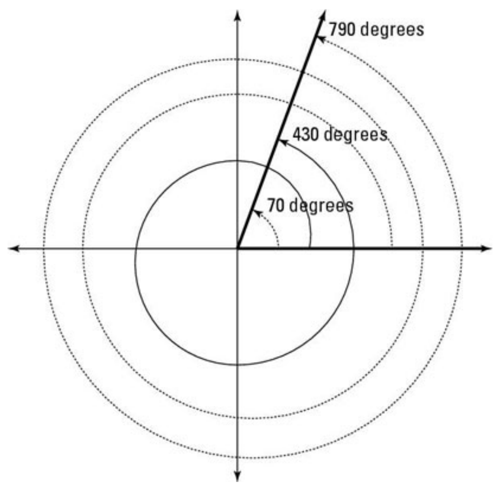
Trigonometric Ratios

These notes are intended as a summary of section 6.1 (p. 466 – 474) in your workbook. You should also read the section for more complete explanations and additional examples.

Coterminal Angles

Angles in standard position that have the same terminal arm are said to be **coterminal angles**. For example, the angles 70°, 430°, and 790° are coterminal.



There are infinitely many angles that are coterminal with a given angle. To determine angles that are coterminal with an angle θ you simply add or subtract multiples of 360°. Thus, the measure of any angle coterminal with θ can be written as:

$$\theta + k \cdot 360^\circ$$

where k is an integer ($k \in \mathbb{Z}$).

Example 1 (sidebar p. 469)

a) Determine the measures of all the angles in standard position between -800° and 800° that are coterminal with an angle of 85° in standard position. Sketch the angles.

b) Write an expression for the measures of all the angles that are coterminal with an angle of 85° in standard position.

Reciprocal Trigonometric Ratios

In addition to the three primary trigonometric ratios, there are also three related ratios. These are called the **reciprocal trigonometric ratios**.

- The reciprocal of cosine is called **secant**. $\sec\theta = \frac{1}{\cos\theta}$
- The reciprocal of sine is called **cosecant**. $\csc\theta = \frac{1}{\sin\theta}$
- The reciprocal of tangent is called **cotangent**. $\cot\theta = \frac{1}{\tan\theta}$

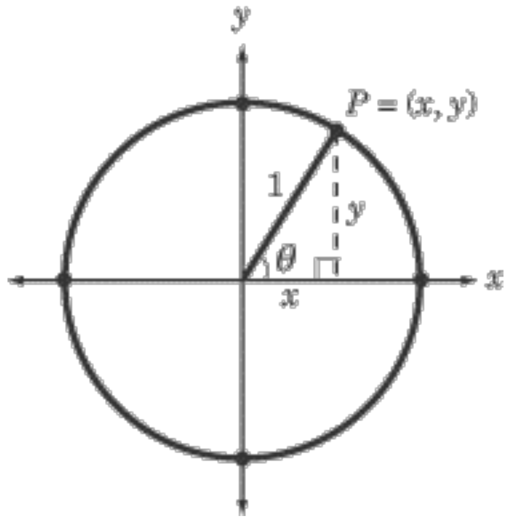
Example 2 (sidebar p. 471)

a) Determine the exact values of the six trigonometric ratios for $-420°$.

b) Determine the approximate values of the six trigonometric ratios for $586°$, to the nearest thousandth.

The Unit Circle

The unit circle is a circle with vertex at the origin and a radius of 1. Its equation is $x^2 + y^2 = 1$.

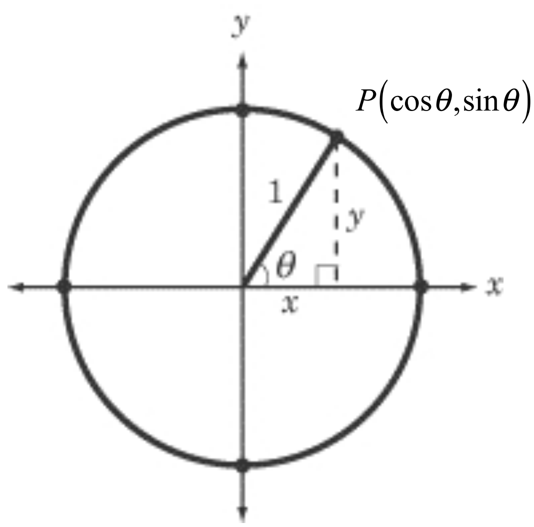


If the terminal point $P(x, y)$ of an angle θ in standard position lies on the unit circle, then we can construct a right angled triangle (as shown above) with legs *x* and *y*, and a hypotenuse of 1. Using this triangle, we can determine values of the six trigonometric ratios:

$\cos\theta = x$ $\qquad$ $\sin\theta = y$ $\qquad$ $\tan\theta = \dfrac{y}{x}$

$\sec\theta = \dfrac{1}{x}$ $\qquad$ $\csc\theta = \dfrac{1}{y}$ $\qquad$ $\cot\theta = \dfrac{x}{y}$

Based on these results, the coordinates of P can be rewritten as:



So, for any point P on the unit circle, its coordinates will always be $(\cos\theta, \sin\theta)$. We can use this to determine the exact trigonometric ratios of certain angles in Quadrant 1, or of angles in the other quadrants that have the same reference angle (see Example 2).

Note 1: It is also possible to rewrite the equation of the unit circle in terms of the trigonometric ratios.

$$x^2 + y^2 = 1$$
$$(\cos\theta)^2 + (\sin\theta)^2 = 1$$

$$\boxed{\cos^2\theta + \sin^2\theta = 1}$$

This relationship is easily verified. Using the special triangles, answer the following questions:

1. What is the exact value of $\sin 60°$?

2. What is the exact value of $\cos 60°$?

3. What is the exact value of $\cos^2 60° + \sin^2 60°$?

The beauty of this relationship is that it is true for **all** values of θ.

Note 2: Another interesting relationship arises from our understanding of the unit circle. We saw above that

$$\cos\theta = x \qquad \sin\theta = y \qquad \tan\theta = \frac{y}{x}$$

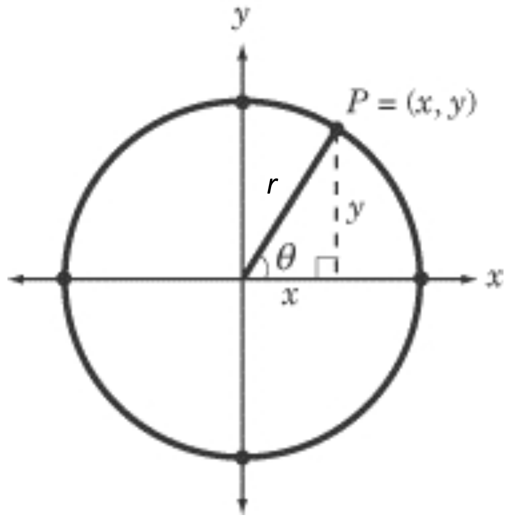
Give these relationships, notice that:

$$\tan\theta = \frac{y}{x}$$

$$\boxed{\tan\theta = \frac{\sin\theta}{\cos\theta}}$$

Terminal Point not on the Unit Circle

Consider a terminal point $P(x, y)$ that does not lie on the unit circle. Even though it is not on the unit circle, we can consider it to lie on a circle with radius *r*. The equation of this circle will be $x^2 + y^2 = r^2$.



Much as we did earlier, we can use this to determine trigonometric ratios for θ. For any angle θ in standard position, with a terminal point $P(x, y)$ on a circle, radius *r*,

$$\cos\theta = \frac{x}{r} \qquad \sin\theta = \frac{y}{r} \qquad \tan\theta = \frac{y}{x}$$

$$\sec\theta = \frac{r}{x} \qquad \csc\theta = \frac{r}{y} \qquad \cot\theta = \frac{x}{y}$$

Example 3 (sidebar p. 472)

$P(-1, -4)$ is a terminal point of angle θ in standard position. Determine the exact values of the six trigonometric ratios for θ.

Example 4 (sidebar p. 473)
Suppose $\sec\theta = 4$.

a) Determine the exact values of the other trigonometric ratios for $0° \le \theta \le 180°$.

b) To the nearest degree, determine possible values of θ in the domain $-360° \le \theta \le 360°$.

Homework: #4, 5, 7 – 11 in the exercises (p. 474 – 480). Answers on p. 481.